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P-WAVE ΛN-ΣN COUPLING AND THE SPIN-ORBIT SPLITTING OF ${}^{9}_{\Lambda}$Be

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We reexamine the spin-orbit splitting of ${}^{9}_{\Lambda}$Be excited states in terms of the SU_6 quark-model baryon-baryon interaction. The previous folding procedure to generate the $\Lambda\alpha$ spin-orbit potential from the quark-model ΛN LS resonating-group kernel predicted three to five times larger values for $\Delta E_{\ell s} = E_x(3/2^+) - E_x(5/2^+)$ in the model FSS and fss2. This time, we calculate $\Lambda\alpha$ LS Born kernel, starting from the LS components of the nuclear-matter G-matrix for the Λ hyperon. This framework makes it possible to take full account of an important P-wave ΛN–ΣN coupling through the antisymmetric $LS^{(-)}$ force involved in the Fermi-Breit interaction. We find that the experimental value, $\Delta E^{\rm exp}_{\ell s} = 43 \pm 5$ keV, is reproduced by the quark-model G-matrix LS interaction with a Fermi-momentum around $k_F = 1.0$ fm^{-1}, when the model FSS is used in the energy-independent renormalized RGM formalism. On the other hand, the model fss2 gives too large splitting of almost 200 keV, owing to the uncanceled contribution of the scalar-meson exchange LS components.

Keywords: Quark-model baryon-baryon interaction; spin-orbit splitting of Λ hypernuclei

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1. Introduction

In view of rich experimental data accumulated for the light Λ-hypernuclei, [1,2] it is important to examine if various models of the fundamental hyperon-nucleon (YN) interactions can reproduce these experimental data or not. For few-body systems, this program is most reliably carried out by detailed Faddeev calculations for the hypertriton (${}^{3}_{\Lambda}$H), [3] ${}^{4}_{\Lambda}$H and ${}^{4}_{\Lambda}$He, [4] using some versions of the Nijmegen models [5] and Jülich potentials. [6] The knowledge of the ΛN interaction learned from these calculations, however, is mainly about the central part of the interaction and features of the ΛN–ΣN coupling of the ${}^3S_1 + {}^3D_1$ state due to the one-pion exchange tensor force. For the p-shell Λ-hypernuclei, some kinds of models inevitably need to be assumed so far, to connect properties of the Λ-hypernuclei and the underlying YN interactions. For example, the small spin-orbit (ℓs) splitting commonly observed in many of the light Λ-hypernuclei [2] is typically manifested in the excited states of ${}^{9}_{\Lambda}$Be, for which a simple $\Lambda+\alpha+\alpha$ three-cluster model is usually employed with approriate $\Lambda\alpha$ and $\alpha\alpha$ potentials. [7] In the framework of this model, the origin of the ℓs splitting for the $5/2^+$ and $3/2^+$ excited states is the spin-orbit potential between Λ and one of the α clusters, which is known to be very small due to the strong cancellation between the symmetric (LS) and antisymmetric ($LS^{(-)}$) LS forces of the ΛN interaction.

In our previous study of the ${}^{9}_{\Lambda}$Be spectrum, [8] we have carried out the $\Lambda\alpha\alpha$ three-cluster Faddeev calculation, trying to reproduce the very small ℓs splitting of the $5/2^+$ and $3/2^+$ excited states, $\Delta E^{\rm exp}_{\ell s}=43\pm 5$ keV, [2] experimentally observed. As a first step, Ref. 8 directly used the quark-model (QM) ΛN LS resonating-group kernel (RGM kernel) to generate the $\Lambda\alpha$ LS potential by a simple procedure of the α-cluster folding. In this approach, the QM ΛN LS interaction of FSS or fss2 predicts 3 to 5 times larger values for $\Delta E_{\ell s}$, which is not much improved in comparison with the results of Nijmegen simulated potentials. [7] It was pointed out in Ref. 8 that a further reduction is possible in the model FSS, if one can properly take into account the short-range correlation of the P-wave ΛN-ΣN coupling by the $LS^{(-)}$ force. This was conjectured through the analysis of the Scheerbaum factors for the single-particle (s.p.) spin-orbit potentials, calculated in the G-matrix formalism.

2. Calculational Procedure

Following the above suggestion, we here generate $\Lambda\alpha$ LS Born kernel from the LS component of the nuclear-matter G-matrix for the Λ hyperon. Our calculation consists of the following three steps.

1. Solve the G-matrix equation for the Λ-hyperon in symmetric nuclear matter with an appropriate Fermi momentum k_F and determine the s.p. potentials for N, Λ and Σ. [9,10]
2. The LS components of the ΛN G-matrices with definite momenta K and starting energies ω are converted to the $\Lambda\alpha$ Born kernel by the folding procedure recently developed for the $\Lambda\alpha$ system. [11]
3. Solve $\Lambda\alpha\alpha$ three-cluster system in the Faddeev formalism for composite particles. [8]

We generate $\Lambda\alpha$ LS Born kernel from our QM baryon-baryon interactions, FSS and fss2. [12] For the $(0s)^4$ α-cluster folding, a new method developed in Ref. 11 is used to derive the direct and knock-on terms of the interaction Born kernel from the ΛN G-matrix, with explicit treatments of the nonlocality and the center-of-mass motion between Λ and α. The G-matrix calculations are carried out by assuming a constant value of the Fermi momentum, $k_F=1.07,\ 1.20$, and $1.35~\text{fm}^{-1}$ (the normal saturation density ρ_0), since the local density approximation does not seem to work in light nuclear systems. The G-matrix equation is solved for the energy-

Table 1. The Scheerbaum factor S_Λ for symmetric nuclear matter and the ℓs splitting of the ${}^{9}_{\Lambda}$Be excited states predicted by the quark–model G-matrix $\Lambda\alpha$ LS Born kernel. In the last column, "ΛN Born" implies the previous results, [8] in which the ΛN single-channel RGM kernel is used for the S_Λ calculation and the α-cluster folding.

	ρ/ρ_0 k_F (fm^{-1})	0.5 1.07	0.7 1.20	1 1.35	ΛN Born –
G-matrix	fss2 (cont)	-11.8	-12.1	-12.3	-10.9
S_Λ (MeV fm^5)	FSS (cont)	-4.1	-5.2	-6.3	-7.8
Faddeev	fss2 (cont)	206	216	223	198
$\Delta E_{\ell s}$ (keV)	FSS (cont)	55	85	114	137
$\Delta E^{\rm exp}_{\ell s}$ (keV)			43 ± 5		

independent QM baryon-baryon interaction, by using the renormalized RGM kernel, [13] and the continuous prescription for intermediate spectra. A similar procedure of the renormalized RGM is also used for the microscopic $\alpha\alpha$ interaction, [14] for which the Pauli forbidden states between the two α-clusters are completely eliminated in the three-cluster RGM formalism using the two-cluster RGM kernels.

3. Results and Discussion

Table 1 shows the ℓs splitting of the ${}^{9}_{\Lambda}$Be excited states, predicted by the $\Lambda\alpha\alpha$ Faddeev calculations, using the QM G-matrix $\Lambda\alpha$ LS Born kernel. The Scheerbaum factor S_Λ is also listed to indicate the strength of the spin-orbit potentials of the Λ hyperon in symmetric nuclear matter. The Fermi momenta $k_F = 1.07,\ 1.20$, and 1.35 fm^{-1} correspond to the densities $\rho = 0.5\,\rho_0,\ 0.7\,\rho_0$, and ρ_0, respectively, with $\rho_0 = 0.17$ fm^{-3} being the normal saturation density. The final values for the ℓs splitting of the $5/2^+$ and $3/2^+$ excited states are $\Delta E_{\ell s} = 55$ - 114 keV for FSS and 206 - 223 keV for fss2, depending on the k_F values in the range of 1.07 - 1.35 fm^{-1}. A smaller k_F value gives a smaller ℓs splitting. If we compare these results with the experimental value $\Delta E^{\rm exp}_{\ell s} = 43 \pm 5$ keV, we find that the model FSS can reproduce the experimental value if the k_F value around 1.02 fm^{-1} is used. We find the strong cancellation between the LS and $LS^{(-)}$ forces taking place in the QM Fermi-Breit interaction by the P-wave ΛN–ΣN coupling in the ${}^1P_1$–${}^3P_1$ state, when the G-matrix equation is solved especially in low-density nuclear matter. This is most prominently exhibited in the model FSS. The spin-orbit contribution from the effective-meson exchange potentials in fss2 does not lead to the sufficiently small ℓs splitting of the Λ hyperon, since the scalar-meson exchange LS force contains only the ordinary LS and does not produce the $LS^{(-)}$ force.

4. Summary

We have carried out $\Lambda\alpha\alpha$ Faddeev calculations by employing the $\Lambda\alpha$ LS Born kernel generated from the LS components of the nuclear-matter G-matrix for the Λ

hyperon. One of our SU_6 QM baryon-baryon interactions, FSS, can reproduce the very small ℓs splitting of ${}^{9}_{\Lambda}$Be excited states, $\Delta E^{\rm exp}_{\ell s} = 43 \pm 5$, when an appropriate k_F value corresponding to almost half of the normal saturation density is employed in the G-matrix calculation. The explicit value of k_F depends on the model construction even within the framework of the $\Lambda\alpha\alpha$ cluster model; $k_F = 1.02$ fm^{-1} for the model FSS, when the energy-independent renormalized RGM kernels are used for the $\alpha\alpha$ RGM kernel and for the QM baryon-baryon interaction. On the other hand, the model fss2 gives too large splitting of almost 200 keV, which is traced back to the uncanceled contribution of the scalar-meson exchange LS components. An essential ingredient of the present formalism is to take into account an important P-wave ΛN–ΣN coupling through the antisymmetric $LS^{(-)}$ force involved in the Fermi-Breit interaction. The present results indicate that the spin-orbit contribution from the effective meson-exchange potentials in fss2 needs to be improved to reproduce the small spin-orbit interaction of the Λ hyperon in the nuclear medium. A new model for the ΛN interaction with consistent central and LS components is strongly desired.

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